Navigation Sensors and Systems

Reference used:
Titterton, D.H., and J.L. Weston 1997. Strapdown inertial navigation technology. Peter Peregrinus and IEE, London.

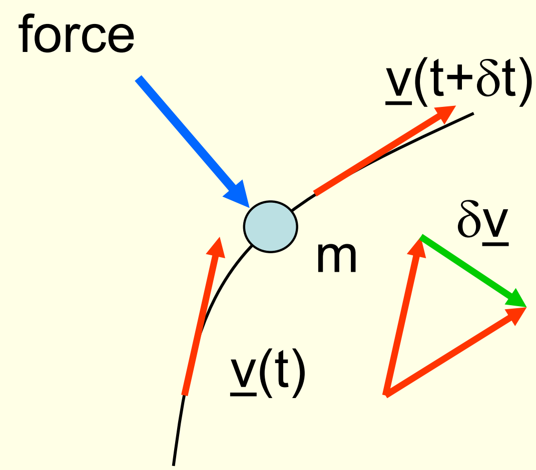
What is Inertial Navigation?

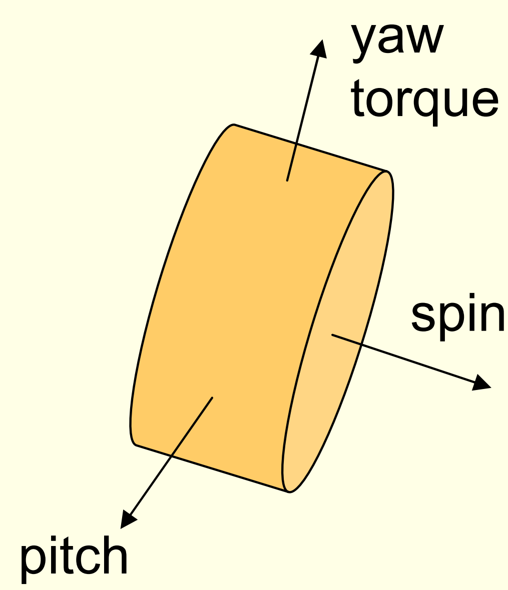
- Navigation: Locating oneself in an environment, e.g., dead-reckoning.
- Inertial: use of Newtonian mechanics:
  - Body in linear motion stays in motion unless acted on by an external force, causing an acceleration:

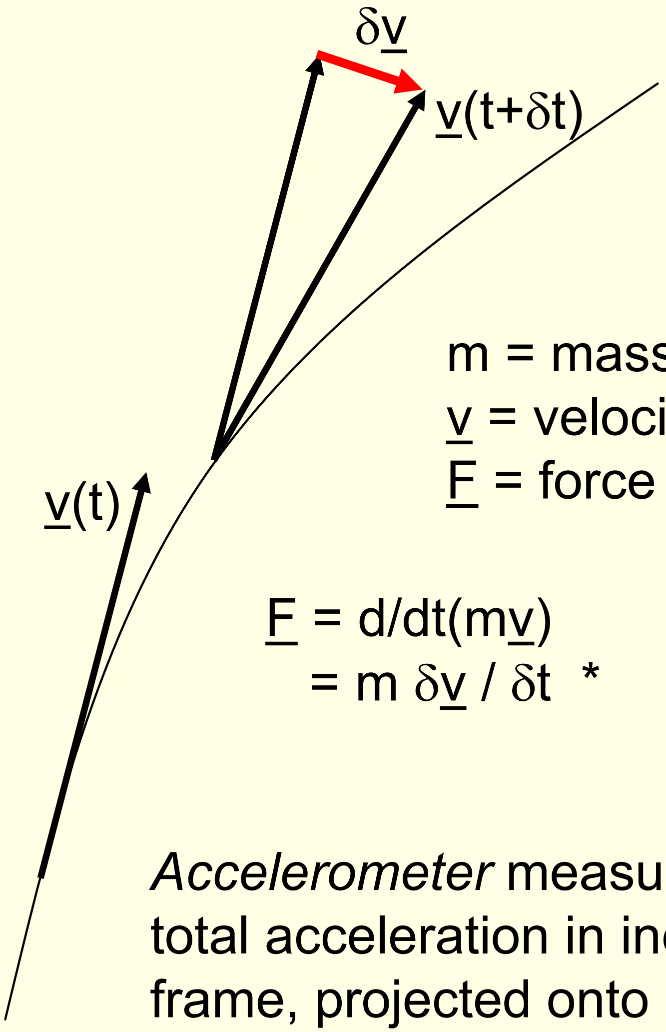
    $\underline{f} = d(m\,\underline{v})/dt \rightarrow m\,d\underline{v}/dt$ (* if $dm/dt = 0$!)
  - *A mechanical accelerometer is effectively <u>a load cell</u>.*
  - Rotational velocity is given by a gyroscopic effect:

    $\underline{\tau} = d\,(J\,\underline{\omega})\,/dt$ or

    yaw torque = J_{spin} X spin_rate X pitch_rate
  - *A mechanical rate gyro is effectively a <u>gyroscope with a load cell.</u>*





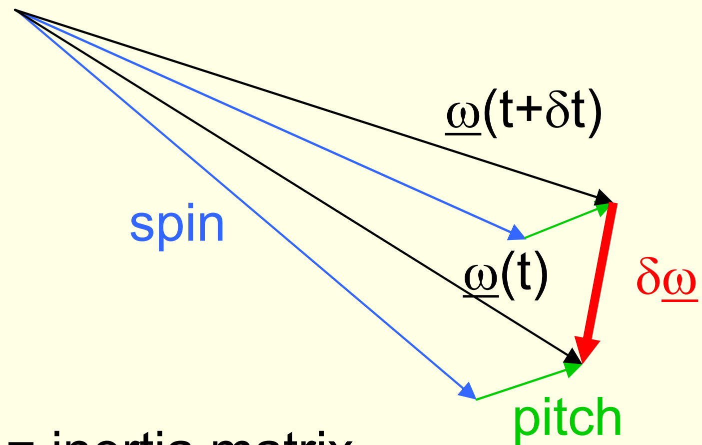


m = mass
$\underline{v}$ = velocity vector
$\underline{F}$ = force vector

$\underline{F}$ = d/dt(m$\underline{v}$)
= m $\delta\underline{v}$ / δt *

Accelerometer measures total acceleration in inertial frame, projected onto platform frame.

Includes, e.g., centrifugal effect, and $\underline{\text{radius}}$ x d$\underline{\omega}$/dt



J = inertia matrix
$\underline{\omega}$ = rotation rate vector
$\underline{\tau}$ = torque vector

$\underline{\tau}$ = d/dt(J $\underline{\omega}$)
= J $\delta\underline{\omega}$ / δt *

Rate gyro measures platform-referenced angular rates:
p (roll rate)
q (pitch rate)
r (yaw rate)

What does accelerometer give? Sum of actual linear acceleration at sensor PLUS projection of gravity

Suppose a 2D sensor is inclined at angle θ. Then measurements are:

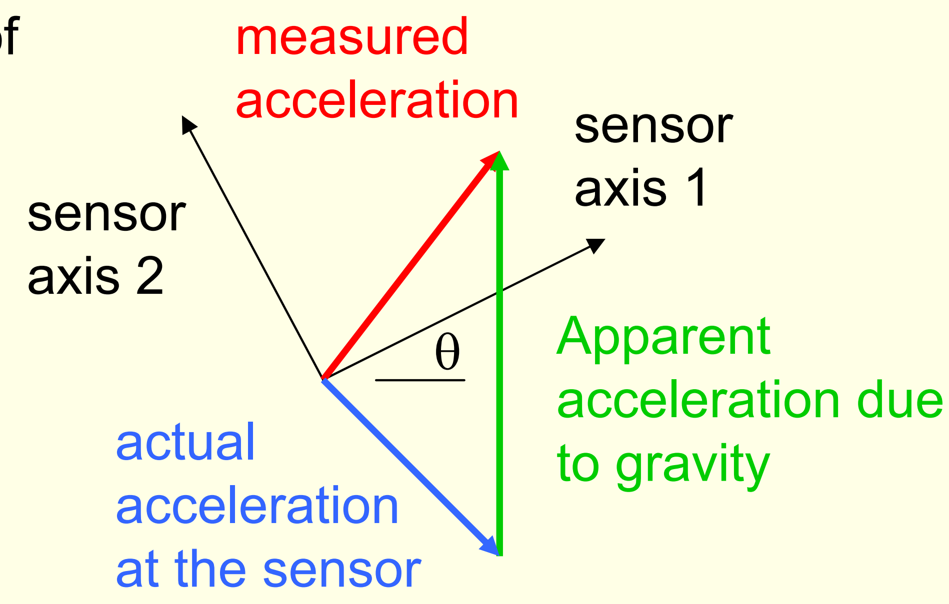
$m_1 = dv_1/dt + g \sin \theta$
$m_2 = dv_2/dt + g \cos \theta$

Case of three sensors:

$m_1 = dv_1/dt + g R_1(\phi,\theta,\psi)$
$m_2 = dv_2/dt + g R_2(\phi,\theta,\psi)$
$m_3 = dv_3/dt + g R_3(\phi,\theta,\psi)$

OR

$\underline{m} = d\underline{v}/dt + g \underline{R}(\phi,\theta,\psi)$



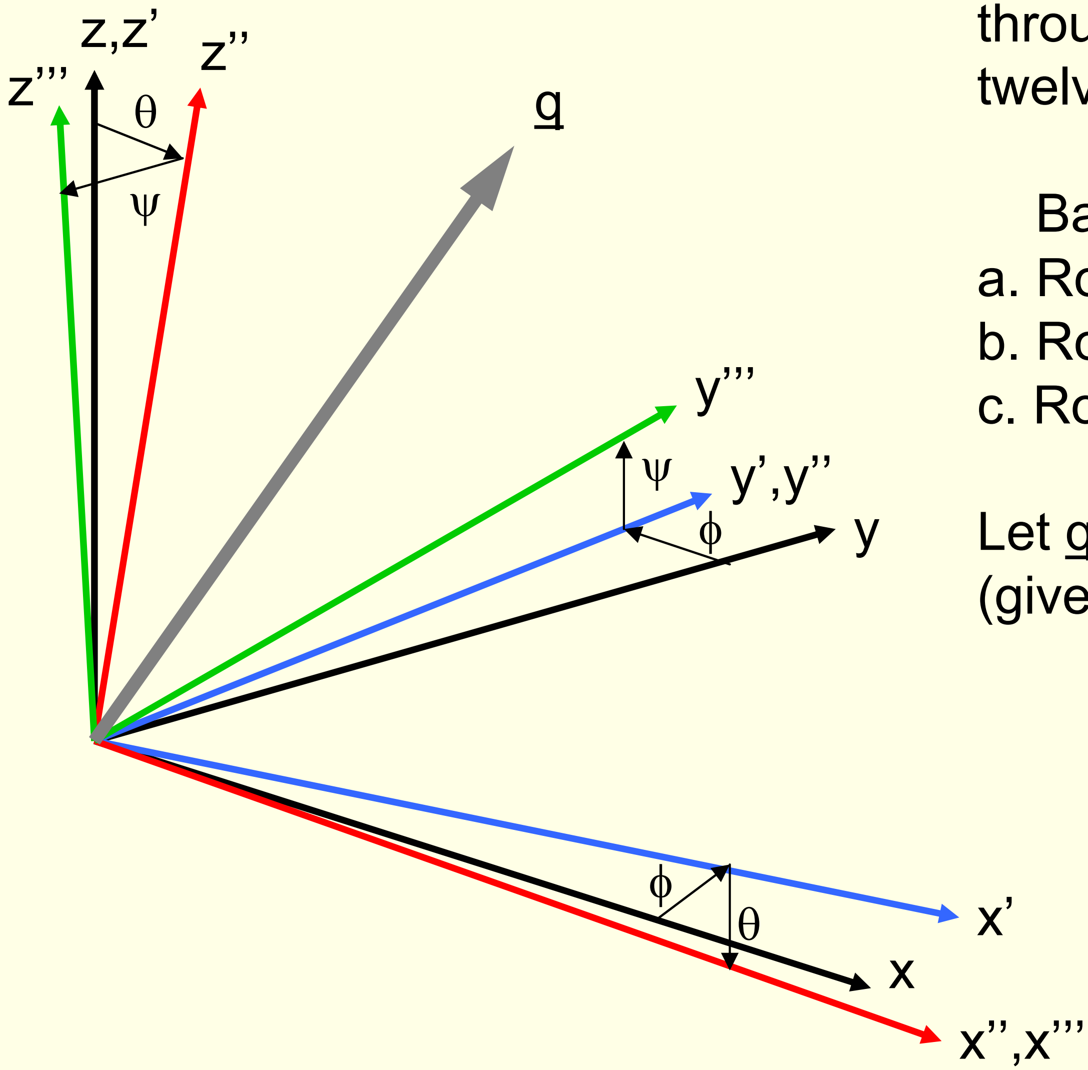
<u>Suppose you integrate:</u>

$\underline{v}$ is sensor referenced velocity, related to velocity in a base frame by

$\underline{v}_b = R^T(\phi,\theta,\psi)\underline{v}$

$[\phi,\theta,\psi]$ are Euler angles; they completely define the attitude of the sensor

Coordinate Frames



Objective: to express a vector q in various frames of reference

Any frame can be transformed to another frame through a translation and a rotation through three Euler angles [ϕ,θ,ψ]. One of twelve possible sequences is:

Base frame is [x,y,z]
a. Rotate about z by ϕ to give [x',y',z']
b. Rotate about y' by θ to give [x'',y'',z'']
c. Rotate about x'' by ψ to give [x''',y''',z''']

Let $\underline{q}$ be given in the base frame – then $\underline{q}'''$ (given in the rotated frame) is:

$$\underline{q}''' = R(\phi,\theta,\psi)\, \underline{q}$$

where R is the *rotation matrix*

Board example!

Rate gyros are *pure* – they give exactly the sensor-referenced rates → can combination of three accelerometers and three rate gyros provide navigation?

Accelerometers contain g projected through the attitude.

Gyros give only angular rate; an integral will drift over time!

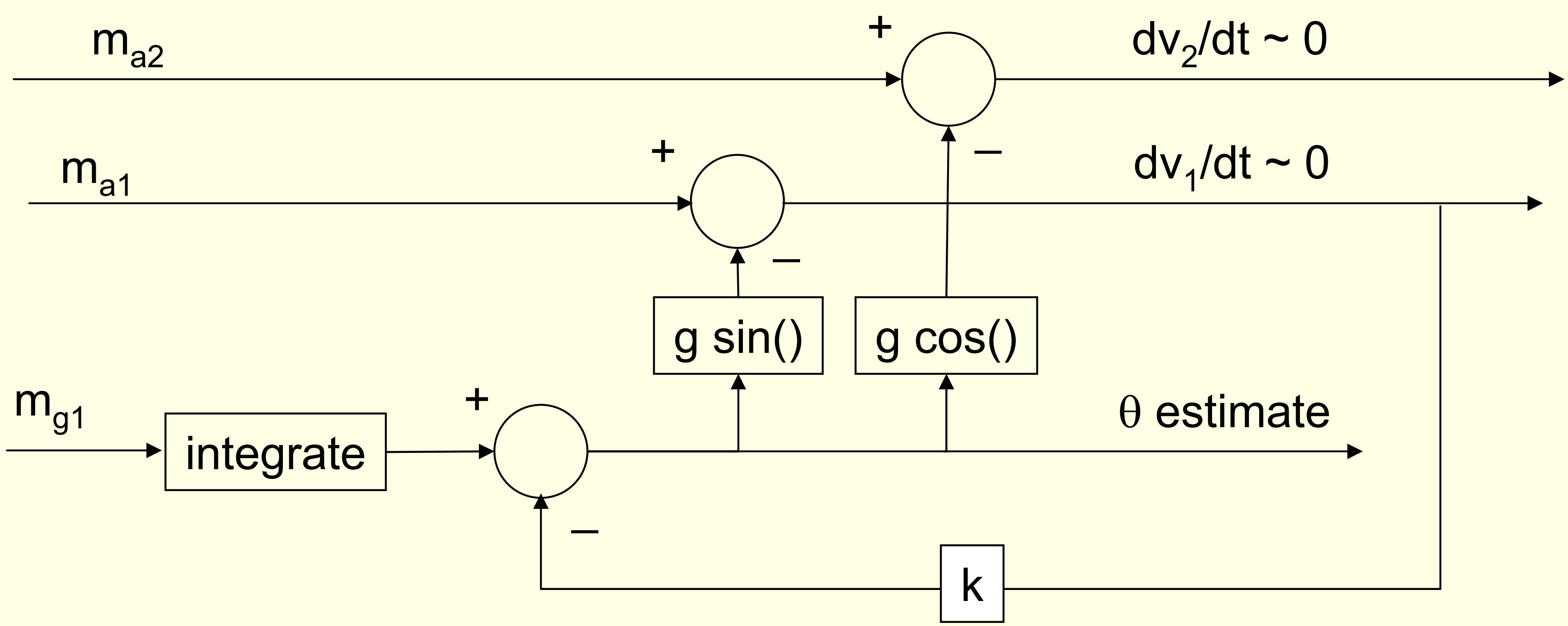
Consider one rate gyro and two accelerometers:

$m_{g1} = d\theta/dt$

$m_{a1} = dv_1/dt + g \sin \theta$

$m_{a2} = dv_2/dt + g \cos \theta$

One procedure for an attitude package (if accelerations are small compared to $g\theta$):



Some Gyro Corrections:

Rotation of the earth:

$\underline{\omega}_E \cos L$

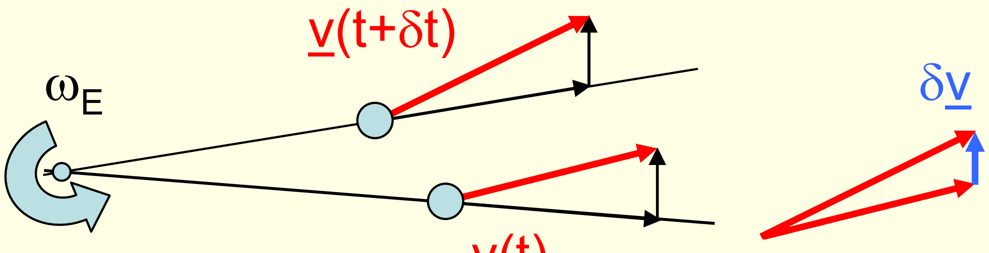
Curvature of the earth:

$\underline{v} / R$

Coriolis acceleration:

$\underline{\omega}_E \times \underline{v}$

L:	lattitude
$\underline{\omega}_E$:	earth rotation vector; magnitude is 0.0042 deg/s
R:	Earth radius, 6400km
$\underline{v}$:	platform velocity



(view from above North Pole)

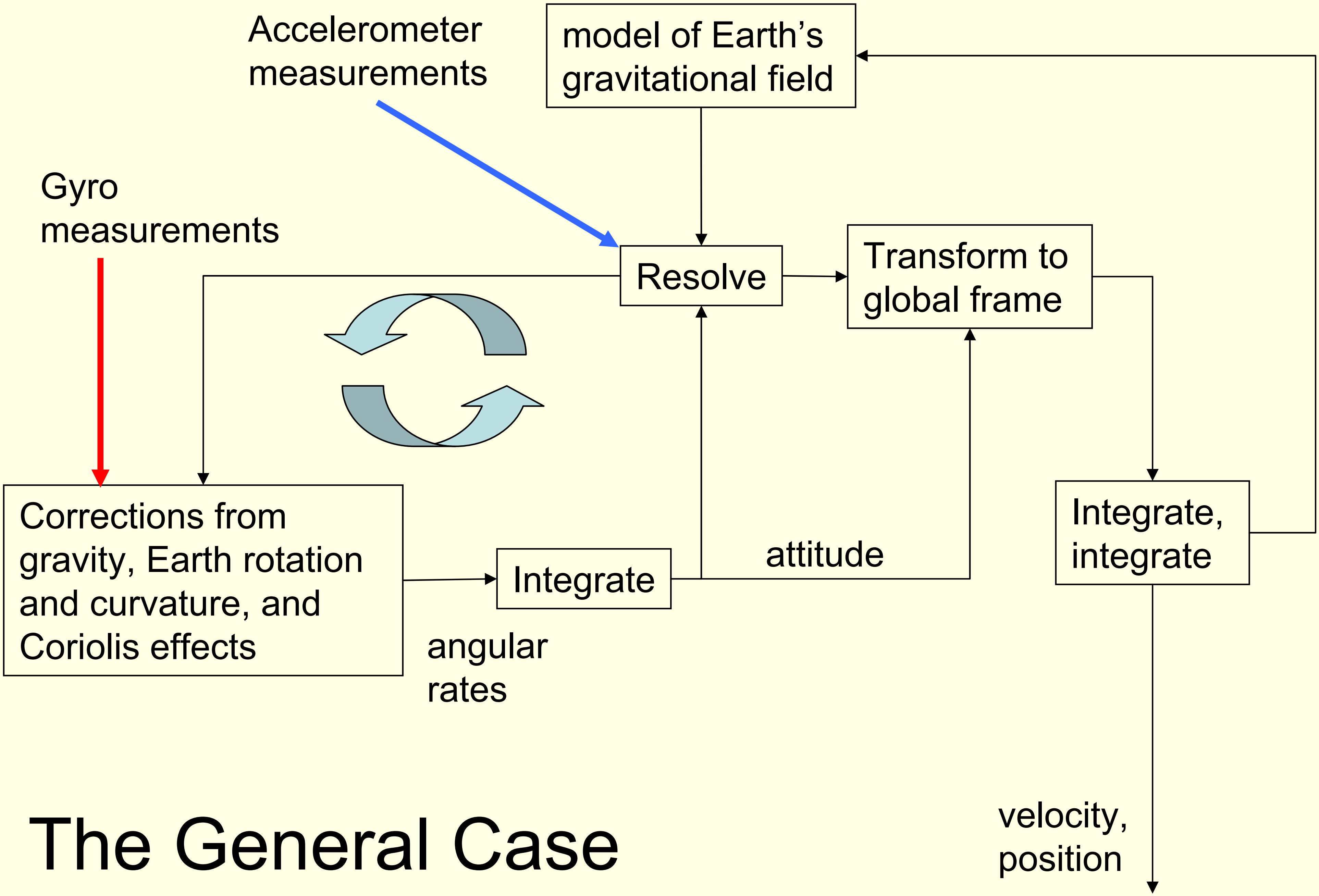
Some Accelerometer Corrections:

Centripetal acceleration due to Earth rotation:

$\omega_E^2 / R \cos L$

Variation of gravity field with lat./long.: e.g.,

$$g(z=0) = 9.780318 * [1 + 0.00530 \sin^2 L - 0.000006 \sin^2 2L]$$



The General Case

Gyroscope Types

- Mechanical: 0.05-20 degrees per hour drift.
- Vibration (e.g., tuning fork) : 360 - 3600 degrees per hour. **Cheap and small!**
- Optical (ring laser): 0.001-10 degrees per hour.
- Optical (fiber optic) : 0.5 – 50 degrees per hour.

Accelerometer Types

Image removed for copyright reasons.
Honeywell HG1700 IMU.

- Displaced spring
- Pendulous mass: 0.1-10 mg bias
- Silicon MEMS: < 25 mg **Small, can be cheap**

Crossbow IMU700:
20 deg/hr fiber optic (3),
9 mg silicon (3)

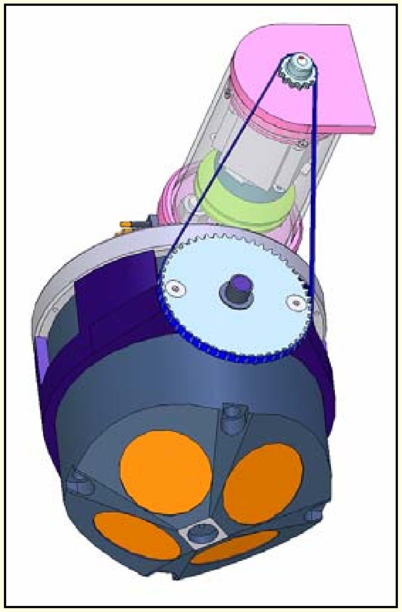
Honeywell HG1700:
1 deg/hr ring-laser (3),
1 mg silicon (3)

Litton LM100 INS:
0.003 degree/hr ring laser
0.025 mg silicon

What is achievable with INS?

The Litton LN100 alone achieves ~1mile/hr drift; depends strongly on errors in initialization.

INTEGRATED NAVIGATION SYSTEM augments the inertial system with complementary sources – i.e., an absolute measurement:



GPS hits (in air only)
Radio beacon (aircraft)
Celestial navigation (clear air only)
Doppler radar (air) or Doppler acoustics (seabed)
Altitude (air) or depth (water)
Range using lasers (air) or acoustics (underwater)
Magnetic field dip angle, relative to a map
Terrain/scene matching, relative to an image database
Etc.

Two Ranging Systems for Positioning

1. GPS: Global Positioning Satellite

- Speed of light or EM waves is 3×10^8 m/s in free space, covering about 30cm in 1ns → a GPS system with 5m precision is achieving time control of all components at the level of 15ns
- *Extremely well-described paths*
- *Extremely accurate clocks on-board*

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GPS satellites orbiting the globe.

- Satellites fire words toward Earth at precise times, which encode their own precise position and trajectory.
- Receiver gets signals from multiple satellites → triangulation → solution in 3-space
- A one-way transmission – from the satellites to your receiver. We need a very good time estimate on the receiver. This is found iteratively, and is part of the "warm-up" time of your receiver.

Interpreting Latitude/Longitude

- Boston is at latitude 42.37^o N,
  longitude 71.03^o W *(approx.)*
- 1 international nautical mile = 1852.00m
- 1 degree of latitude = 60 nm = 111.12 km
- 1 degree of longitude = 60 nm * $\cos(42.37^o)$
  = 44.33 nm = 82.10 km

60 minutes in a degree →

one minute latitude = 1852 m

one minute longitude = 1368 m

60 seconds in a minute, etc.

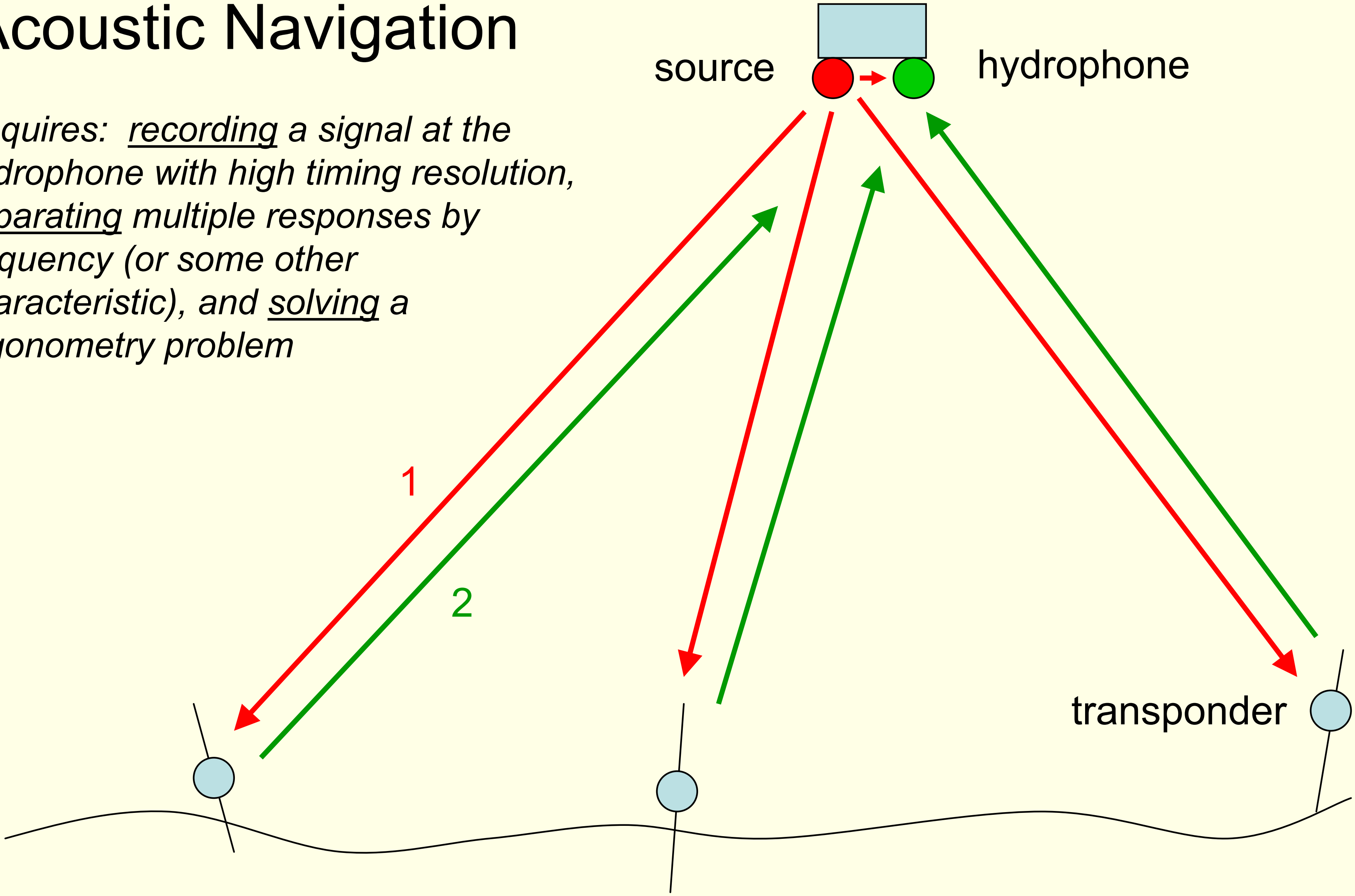
Common format: decimal degrees (DD)* – *a <u>double</u> type

2. Acoustic Ranging

- Similar to GPS; speed of sound in water is ~1450 m/s, so 1m precision requires timing precision around 0.6ms.
- Accuracy limited by spatial variation of sound speed
- Some use of one-way travel times, but two-way systems have been more common to date, e.g., a long-baseline (LBL) system:
  - Vehicle pings using a *source* or *transducer*
  - *Responders* hear it, and ping back with unique frequencies. Responder locations are known to the vehicle
  - Vehicle receives the signals with a *hydrophone*, and measures a set of two-way travel times to each responder → triangulation
- An "inverse" problem: multiple hydrophones on vehicle, but one responder → an ultra-short baseline (USBL) system that gives relative direction and range to target.

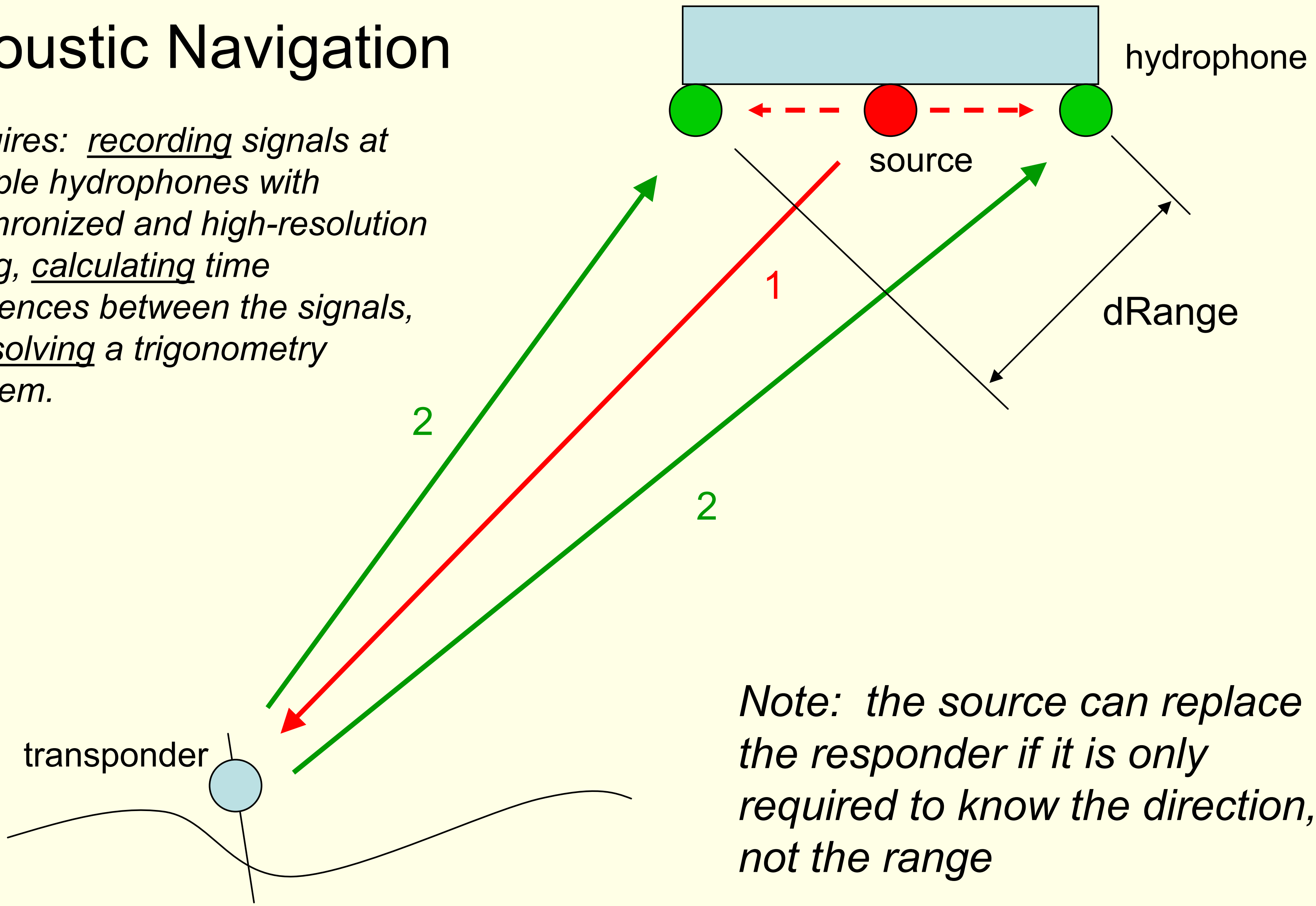
Long-Baseline Acoustic Navigation

Requires: <u>recording</u> a signal at the hydrophone with high timing resolution, <u>separating</u> multiple responses by frequency (or some other characteristic), and <u>solving</u> a trigonometry problem



Ultra-Short Baseline Acoustic Navigation

Requires: <u>recording</u> signals at multiple hydrophones with synchronized and high-resolution timing, <u>calculating</u> time differences between the signals, and <u>solving</u> a trigonometry problem.



Note: the source can replace the responder if it is only required to know the direction, not the range